

Exercise 46Calculate y' .

$$y = \ln \left| \frac{x^2 - 4}{2x + 5} \right|$$

SolutionCalculate y' by using the chain and quotient rules.

$$\begin{aligned} y' &= \frac{d}{dx} \ln \left| \frac{x^2 - 4}{2x + 5} \right| \\ &= \frac{d}{dx} \ln \sqrt{\left(\frac{x^2 - 4}{2x + 5} \right)^2} \\ &= \frac{1}{\sqrt{\left(\frac{x^2 - 4}{2x + 5} \right)^2}} \cdot \frac{d}{dx} \sqrt{\left(\frac{x^2 - 4}{2x + 5} \right)^2} \\ &= \frac{1}{\sqrt{\left(\frac{x^2 - 4}{2x + 5} \right)^2}} \cdot \frac{1}{2} \left[\left(\frac{x^2 - 4}{2x + 5} \right)^2 \right]^{-1/2} \cdot \frac{d}{dx} \left(\frac{x^2 - 4}{2x + 5} \right)^2 \\ &= \frac{1}{\sqrt{\left(\frac{x^2 - 4}{2x + 5} \right)^2}} \cdot \frac{1}{2} \left[\left(\frac{x^2 - 4}{2x + 5} \right)^2 \right]^{-1/2} \cdot \frac{d}{dx} \left(\frac{x^2 - 4}{2x + 5} \right)^2 \\ &= \frac{1}{\sqrt{\left(\frac{x^2 - 4}{2x + 5} \right)^2}} \cdot \frac{1}{2} \left[\left(\frac{x^2 - 4}{2x + 5} \right)^2 \right]^{-1/2} \cdot 2 \left(\frac{x^2 - 4}{2x + 5} \right)^1 \cdot \frac{d}{dx} \left(\frac{x^2 - 4}{2x + 5} \right) \\ &= \frac{1}{\sqrt{\left(\frac{x^2 - 4}{2x + 5} \right)^2}} \cdot \frac{1}{\sqrt{\left(\frac{x^2 - 4}{2x + 5} \right)^2}} \cdot \left(\frac{x^2 - 4}{2x + 5} \right) \cdot \frac{d}{dx} \left(\frac{x^2 - 4}{2x + 5} \right) \\ &= \frac{1}{\left(\frac{x^2 - 4}{2x + 5} \right)^2} \cdot \left(\frac{x^2 - 4}{2x + 5} \right) \cdot \frac{d}{dx} \left(\frac{x^2 - 4}{2x + 5} \right) \\ &= \frac{1}{\left(\frac{x^2 - 4}{2x + 5} \right)} \cdot \frac{d}{dx} \left(\frac{x^2 - 4}{2x + 5} \right) \\ &= \frac{2x + 5}{x^2 - 4} \cdot \frac{d}{dx} \left(\frac{x^2 - 4}{2x + 5} \right) \end{aligned}$$

Therefore,

$$\begin{aligned}y' &= \frac{2x+5}{x^2-4} \cdot \frac{\left[\frac{d}{dx}(x^2-4)\right](2x+5) - \left[\frac{d}{dx}(2x+5)\right](x^2-4)}{(2x+5)^2} \\&= \frac{2x+5}{x^2-4} \cdot \frac{(2x)(2x+5) - (2)(x^2-4)}{(2x+5)^2} \\&= \frac{2x+5}{x^2-4} \cdot \frac{2x^2+10x+8}{(2x+5)^2} \\&= \frac{2x+5}{x^2-4} \cdot \frac{2(x^2+5x+4)}{(2x+5)^2} \\&= \frac{2x+5}{(x+2)(x-2)} \cdot \frac{2(x+4)(x+1)}{(2x+5)^2} \\&= \frac{2(x+4)(x+1)}{(x+2)(x-2)(2x+5)}.\end{aligned}$$